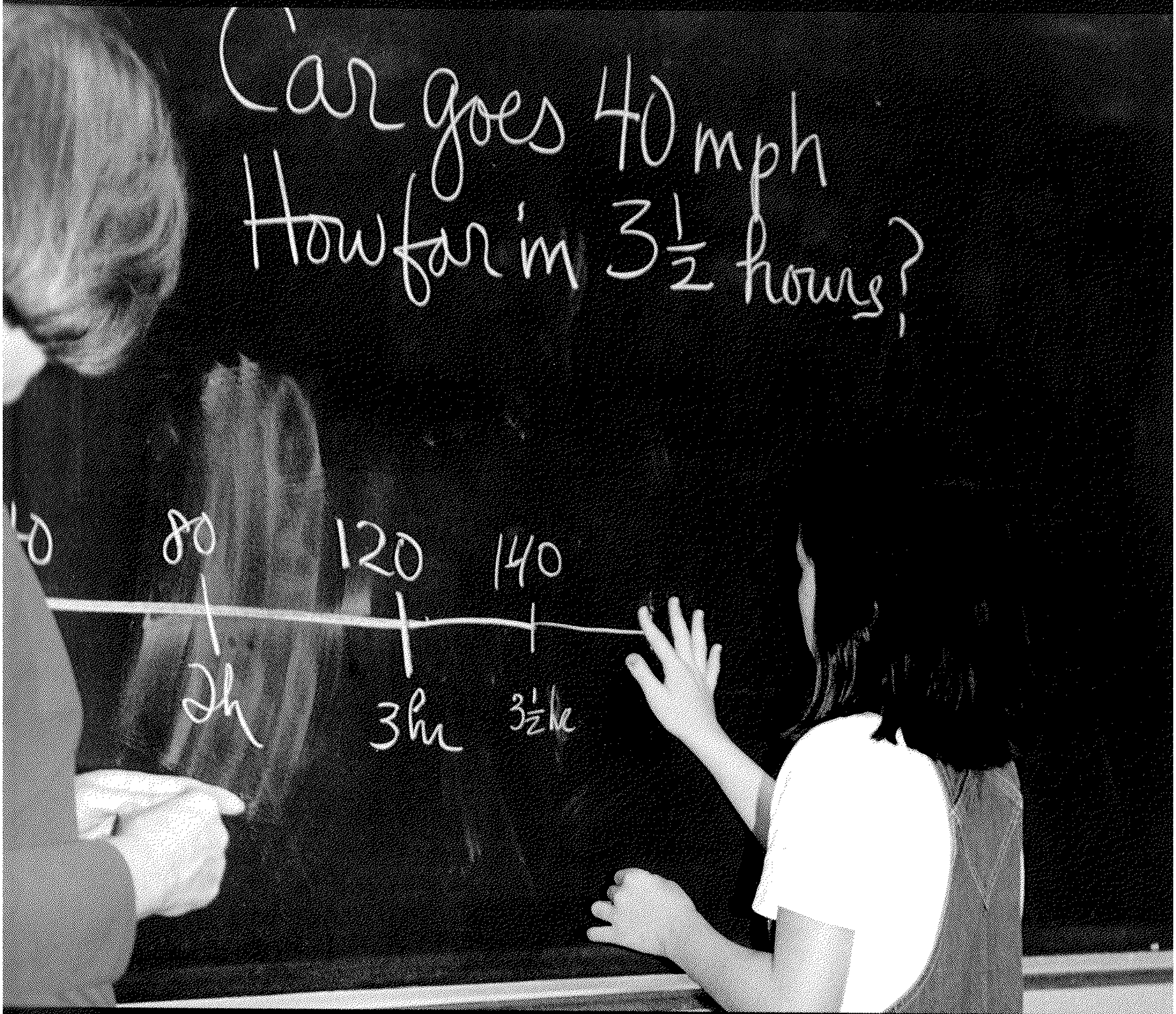
TEACHING PROBLEMS AND THE



PROBLEMS OF TEACHING

MAGDALENE LAMPERT

Teaching Problems and the Problems of Teaching

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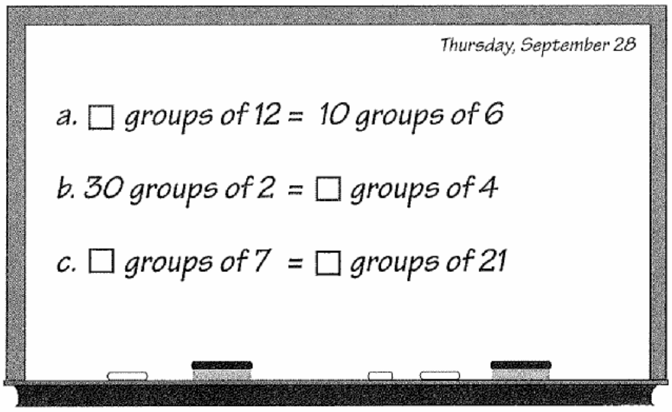
Teaching While Leading a Whole-Class Discussion

The most common image of the teacher at work has her in the front of the room, either addressing the whole class or choosing students to answer questions. In this chapter, I examine problems of practice that arise in this kind of work. Considering actions that involve the teacher in communications with the whole class, I zoom in to investigate who is being taught, what they are being taught, and how they are being taught. I examine the work that this teaching entails. As I interact with the whole class at once, I need to maintain overall coherence while drawing different kinds of individuals into a common experience of the content. I do this, in part, by calling on students to say something that will contribute to the common experience of the class and then constructing responses to what they say. Equally important are the actions I need to take to engage those students who are not verbal participants, such as drawing on the board.

In order to examine practice at this level, I bring a microscopic lens to work that I performed in the whole-group portion of the lesson on September 28. I analyze that work action by action to link teaching individuals with teaching the class as a whole. The talk in this part of the lesson was divided by its attention to the three parts of the task the students had been working on, labeled A, B, and C.



The discussion divides into one segment whose focus was on the mathematics of multiplication as grouping (during which we discussed problems A and B) and a second segment whose focus was on patterns and relationships in ordered pairs of numbers (during which we discussed problem C). Using video records of interactions in each of these segments, I closely examine a ten-minute portion of the discussion of problem A and a smaller portion of the discussion of problem C. I break each segment into many discrete "teaching and studying events" to illustrate the range of teaching problems that arise at this scale and the work entailed in addressing them. (A transcript of the entire large-group discussion is included in appendix A so that the reader can also investigate these events in relation to the continuous flow of talk during the discussion.)

In the teaching events that I examine here, the mathematical scope is wide, and the relationship in focus is my relationship with the class as a whole. In addition to the problems of teaching the mathematical content, I was trying to teach everyone about mathematical discourse, about how to participate in a conversation with twenty-six other people, and about themselves as capable doers of the task and thinkers about the ideas in it. Through these wide social lenses I analyze teaching in small units of time to see how these broader efforts play out in each moment of interaction. This microscopic analysis is at times tedious and complex. But it reveals that each word and gesture the teacher uses has the potential to support the study of mathematics for all students, or not. The labels I have placed on each event identify the actions a teacher takes to support studying by the class as a whole while interacting with individuals. These actions are intended to create opportunities for everyone in the class to study something. Particular students may or may not take advantage of these opportunities, but my work is to make them broadly available.

Teaching in the Moment so the Whole Class Can Study

I guided a discussion for about ten minutes that began with one student's speculation about the solution to problem A:

$$\square \text{ groups of } 12 = 10 \text{ groups of } 6$$

The studies of the class that I built from this problem were organized around an exchange with the first student who volunteered an assertion, Richard. I asked him to explain his solution and I recorded parts of his explanation on the board, inserting my own mathematical commentary. I involved other students in supporting and furthering Richard's thinking. As I responded to what students said and inserted my own agenda, he reevaluated his original assertion. The nature of the discourse shifted around including both rehearsals of multiplication tables and reasoning about grouping. I used this talk to assemble an argument for why Richard's assertion was wrong. After he reevaluated what he said, I made another incorrect assertion about what might go in the box and invited students to evaluate it. Then we moved on slowly, almost painstakingly, to construct a correct assertion about what number would go in the box and why. I guided this construction in a way that kept the talk strongly linked to the representation of multiplication in terms of grouping and regrouping and the idea of multiplication as repeated addition. Several students made use of the diagrams I made on the board during the interaction with Richard to support their assertions.[1]

Although the focus in the following analysis is on what I did to teach, the actions that I took were constructed in concert with my observations of what students did to study and what they were studying as individuals, dynamically constituted groups, and as a whole. The topics of our work together included talk about conventional mathematical content as well as aspects of mathematical practice and of the practice of "studenting"—or learning how to learn—in this classroom setting. In what follows, I focus on the details in my speech acts because particular words and intonations are some of the most important tools I can use in this setting to maintain continuity and coherence for the class while responding to individual students.

Teaching and Studying Event #1: Teacher Formulating and Asking a Question to Begin the Discussion

Lampert: [referring to the first problem] Okay, who has something to say about A?

By asking this opening question in the way I do, I make use of an opportunity to teach students that mathematical talk can have a broad range, and not just be about right and wrong answers to teachers' questions. With this particular choice of words, I open the floor to students for reflection. They can answer my question whether or not they think they have solved the problem. "Something to say" here *can* include the "answer." That is what students are likely to expect the teacher to ask for. But students might also legitimately respond by saying "I finished it," by making assertions pointing toward the mathematics like "that was the easiest one," or with other kinds of commentaries on the work. As students contemplate whether and how to respond to my question, they must interpret what is meant, in the context of this discussion by "something to say."

Teaching and Studying Event #2: Teacher Calling on a Particular Student to Answer

Lampert: Richard?

Several students had raised their hands. Ellie, Connie, Shahroukh, Candice, Enoyat, and Leticia all seemed eager to answer. Because I knew what many students had been doing in the first part of class as they worked independently with their peers, I could exercise the option to call on someone in the class to get a particular piece of mathematics on the table. I decided to call on Richard, even though he had not volunteered to speak. I used my choice of whom to call on to get a particular piece of mathematics up for consideration. I could then direct other members of the class to examine the initial speaker's mathematical work, which would make it possible for them to both study the topic and engage in the practice of mathematical communication.

So why did I call on Richard? How and what and who could I teach by calling on *him*? What would I make available for students to study? And how? In my journal, after the lesson, I wrote about my worries about Richard and another student, Jumanah, doing computations randomly without thinking about the appropriateness of what they were doing to the problem context.

I don't understand at all how Jumanah is thinking → she puts numbers in the spaces in ~~here~~ her notebook that have no conventional relationship to the problem being posed, and she is not at all verbally expressive. In her case, from what little evidence I can gather, as well as in the case of Richard, there seems to be a "put these numbers together in <u>some</u> way," vs. any attempt to see meaning in what is happening.

I called on Richard because I wanted to teach him and others in the class that everyone would indeed be asked to explain their thinking publicly. I also wanted to teach everyone that what they said would be expected to be an effort to make mathematical sense. (As we see in the following parts of the lesson, at this point in the year their efforts to make a sensible explanation would be scaffolded, sometimes heavily; students would not be expected to do this entirely independently, or even to know what is meant by an "explanation.")

One thing students in the class can study as I make my choice to call on Richard is that I take this action in the face of several students' raised hands.

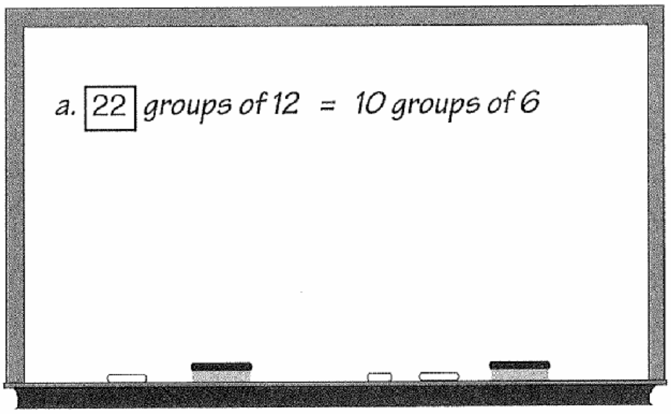
Many would no doubt make their own conjectures about why I called on Richard and why I did not call on someone else. They would continue to conduct experiments to learn more about how to get called on or not, depending on their purposes.

Teaching and Studying Event #3:
Student Asserting and Teacher Repeating His Assertion

When Richard responds, he has the opportunity to study how his teacher and his peers will respond to his assertion.

Richard: I think that if, A, is twenty-two. Groups.

Lampert: [addressing the whole class while writing "22" in the empty box on the board and pausing while stressing the word "of"] Okay, so twenty-two groups OF twelve equals ten groups OF six [pointing to what is on the chalkboard].



In this exchange Richard volunteers an answer. He uses the words "I think" to preface his assertion. I fill the empty box on the chalkboard with his assertion, revoicing it in terms of the problem structure. I repeat his answer in the form I had designed for the problem and write it on the board. By recording Richard's conjecture on the board, I intend to teach my students that any assertions they make would be taken as a serious indication of what they thought was a reasonable solution to the problem as posed (whether or not that is how they intended them to be taken).

Teaching and Studying Event #4:
Teacher Asking a Student to "Explain His Reasoning"

Lampert: Can you explain your reasoning about that, Richard?

As my preparation for this lesson indicated, I wanted to get students working on tasks that would engage them in thinking about when it would make sense to use multiplication to solve problems. By asking Richard to explain his reasoning, I initiate a discussion of why he might have done what he did. I was preparing the social framework in which I would ask other students to agree or disagree with him. I did what I did knowing that several students in the class would have already decided that Richard's answer was wrong because I had seen what they did in their notebooks prior to the discussion. I conduct the discussion as if there was a shared assumption that there would be *reasoning* behind any assertion that would explain why it would make sense. Even though Richard's assertion did not seem to make sense, I respond respectfully, hoping to "dignify with pertinent curiosity" his contribution to the discussion.[2]

Throughout this segment, Candice has her hand up, as does Shahroukh. Although we cannot know for sure, patterns of classroom discourse lead me to suspect that the two students who were indicating they wanted to speak wanted me, and others in the room, to know that they would disagree with Richard. I stick with him at this point instead of entertaining their contributions. This choice produces additional work, as I am also responsible for keeping Candice and Sharoukh engaged, even as I suspect that they are not on the same mathematical path as Richard.

Teaching and Studying Event #5: Student Interpreting "Explaining" and Responding

Richard: Because, I timesed twelve and ten. Twelve times ten equals twenty-two.

In his response, Richard seems to confuse addition and multiplication. Although he *says* "Twelve times ten equals twenty two," he seems to have added instead of multiplying. He has correctly added a group of 10 and a group of 12, to arrive at 22, and there is a "10" and a "12" in the expression. That particular computation is not relevant to finding the unknown, however, given that the problem questions how many *groups* of 12 you need to equal 10 *groups* of 6. He has also transposed the 10 and the 12 in a way that does not fit the problem context.

Richard has presented me here with both a problem and an opportunity. The idea that multiplication is about a particular kind of *grouping* was the central focus of my planning for this lesson, and so here I have an opportunity to demonstrate the meaning of the operation by contrast with Richard's interpretation. The placement of the empty box asks, "How many groups of

12?" or "What number times 12?" would be equal to 60. Richard's assertion was "Twelve times ten equals twenty-two." So there are several bits of mathematics to be sorted out here, some of which have to do with the meaning of multiplication and others of which have to do with the different representations of multiplication in words and symbols. The problem is that I must somehow both teach Richard and engage the whole class in worthwhile mathematical activity at the same time.

To respond to Richard's assertion, I initiate activities that will make it possible for all of the students to study the connection between the action of grouping and the arithmetic operation of multiplication. Based on my observations of their notebook work in the first part of class, I choose representations for communicating both what I am trying to say and what I think students are trying to say. These activities are structured both for participation by Richard and for participation by others in the class in studying multiplication. The others in the class I teach in this way may need to study precisely what Richard needs to study—that multiplication means grouping and that "☐ groups of 12" is translated as "what number times 12"—or they may need to study related topics.

After listening to Richard answer that he "timesed twelve and ten," I move the work of the class into the domain of representation, making available several alternatives for study. By choosing not to call on either Shahroukh or Candice, I intended to hold off on any talk about answers other than the one Richard had given. I also tried to push Richard further toward making sense of the problem, demonstrating both to him and to everyone else that this was a possible teacher move in this circumstance.

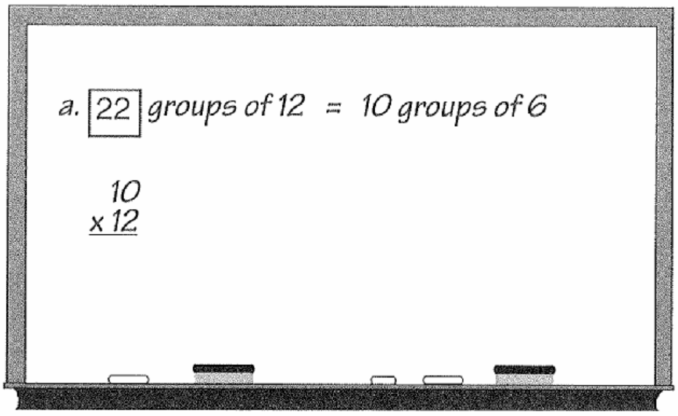
Teaching and Studying Event #6: Teacher Making Representations of Student Talk

My next teaching move is to address the "translation" problem. Here I give Richard and his classmates familiar representations to ponder in relation to the words that Richard used and the problem structure as I presented it.

Lampert: Ten times twelve.

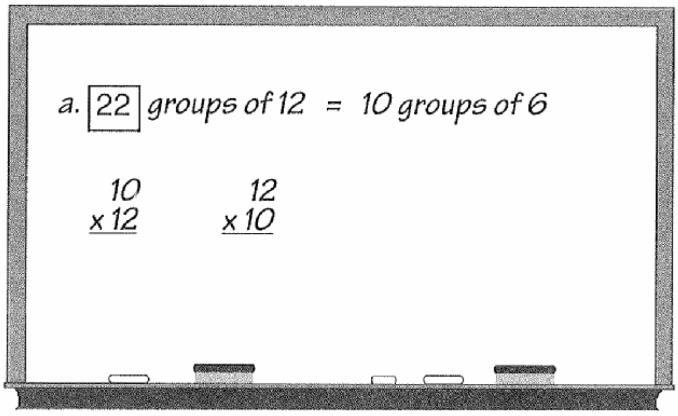
Or—, I'm sorry, twelve times ten is like this.

[I write first the 12, then the times sign (×), then the 10, and then put a line under the 12.]



[to Richard] Is that how you did it?

[to the whole class] Okay, now I want to remind you that this [pointing to the multiplication] means twelve groups of ten [writing on the board, 12 first, then ×, then 10, and a line under the 10]:



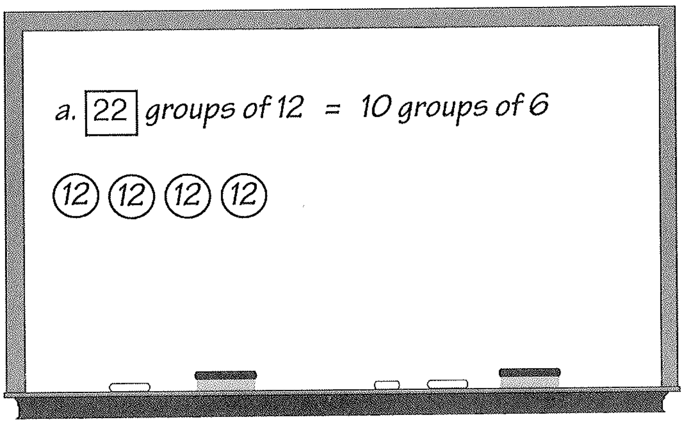
This [pointing to the second multiplication] means ten groups of twelve.

The content of what I am doing here is multidimensional. In one dimension, it has to do with teaching topics, terms, and symbols. I am teaching Richard and the class how to write down a multiplication so that the way they write it down matches what is commonly seen, and how to read it, conventionally. In another dimension, it has to do with teaching the practice of studying mathematics in school. In order to study together, students need to be able to have a shared set of terms and symbols to which their talk refers.

Teaching and Studying Event #7:
Teacher Interpreting Symbols in Terms of an Alternative Representation

As I continued to teach, I continued to mix mathematical content with learning-to-learn content, next showing a more graphic representation of multiplication. I begin the representation only to hint at its usefulness. I refer to the drawings I had guided some students to make in their notebooks during their independent work time to draw in those like Varouna who I knew needed to know more about the meaning of multiplication.

Lampert: One of the things that I came around and did with some people is to draw a picture that would help you to reason about these problems. Twenty-two groups of twelve, you could draw as a twelve, a twelve, a twelve, and so on until you got twenty-two of them [drawing circles around 12s as I talk].

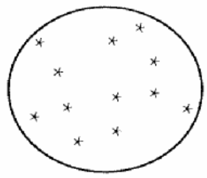


Or you could even put little Xs in the circles like I did yesterday with the paper clips. Twenty-two groups of twelve seems to me like it would be quite a lot of stuff, if I did twenty-two of these [pointing to the four 12s circled on the board].

By choosing to teach Richard in this way, I show him and the other students what "we" do to learn math. Throughout this demonstration, I intend to teach everyone, including Richard, that this is not going to be a class in which the teacher says whether an answer is right or wrong as the first response to any assertion. Rather, I will provide students with tools for reasoning themselves about the appropriateness of their answers.

Here I do mathematics by ranging around various ways to interpret "ten times twelve," making my activity available for observation. I demonstrate another representation for "ten times twelve" to put alongside these words, spoken by Richard. The mathematical work I expect of students (including Richard) is that they study these representations by using them to reason about Richard's assertion. What I did was designed to teach the class that the number they put in the "box" should make sense according to the meaning of the words and symbols we are all using. It was also designed to teach the class that drawing a picture is a way to give words and symbols a publicly available meaning that could then be assessed for its reasonability. I did not tell them the meaning, I created a representation of it for them to look at and interpret. I wanted to teach everyone that "drawing pictures" is not just for those students who could not do computation (i.e., the ones I worked with earlier in the lesson). I also wanted to demonstrate that teaching could happen in both small-group and large-group settings, and that what is taught and studied in these settings is related.

Working at the juncture of words, symbols, and pictures, I began to create a "hybrid" kind of representation. It was a mathematical shortcut, more abstract than making twenty-two of these:

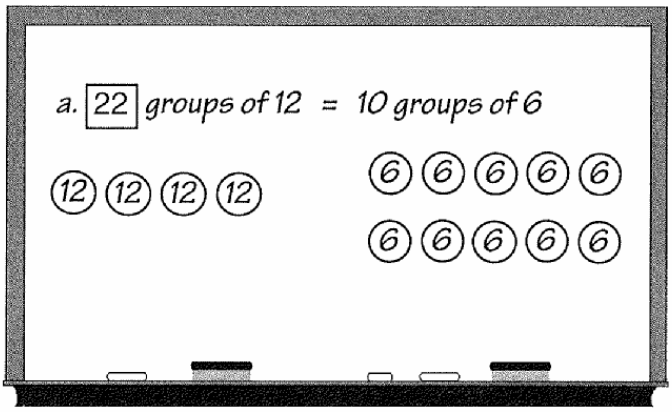


but more concrete than writing "22 x 12" or saying "twenty-two times twelve." By making what Richard called "timesing" into "grouping" I have used language to place my students (including Richard) in a mathematical environment where they can study and evaluate Richard's assertion that "twelve times ten equals twenty-two." I conclude this segment of teaching with an assertion, which I had been laying the groundwork for with my representation: "twenty-two groups of twelve seems like it would be quite a lot of stuff." I leave that assertion hanging in the air. I connect it with *my* reasoning but I do not connect it back to Richard's assertion, seeking to teach Richard and the class that it is up to them to decide what the implications are for judging Richard's assertion. By doing this drawing, I show Richard and the class how drawing a picture could be used to check your assertions, and that there is a particular kind of drawing that would work to check in relation to these kinds of problems. However many groups of 12 we had was supposed to be equal to 10 groups of 6.

Teaching and Studying Event #8:
Teacher Highlighting Patterns to Give Meaning to Multiplication

I leave my representation of "22 groups of 12" unfinished and leave Richard and the others to contemplate what 22 groups of 12 might look like. Moving over to the other side of the equation to represent the groups of 6, I cover up the groups of 12 I had drawn by positioning myself in front of them.

Lampert: But let's look at ten groups of six for a minute [drawing on the board, next to what I had already done]:



Six, six, six, six, six [pointing to the 6s in the circles].

The lesson in the discussion so far could be said to be about how to represent an assertion made in words and symbols in a drawing so as to judge whether it makes sense. With this new representation of groups of 6, I shift to a more fundamental level of teaching about multiplication and shift the focus away from Richard somewhat.

Teaching and Studying Event #9:
Student Interpreting the Public Representation

What I do next is designed to accomplish two aims, to teach students to notice and use patterns in the drawings they produce and to teach them that mathematics is characterized by finding such shortcuts.

Lampert: How do I know that I have ten groups of six there without counting them? Ellie?

Ellie: Because you made um, on the top you made five of them, but, you made five rows on the top and you just made a line exactly like that on the bottom.

Lampert: Okay, that's the story of a mathematical shortcut. I only had to count the first row and then I gave each one of them a partner and that gave me ten groups of six.

Now I give the class a new task: figuring out *my* reasoning. I am using patterns to design and justify a strategy for calculating a total instead of counting "by ones." I am demonstrating what my strategy gets me in this problem context. I challenge the class to imagine and describe what I did. Several students volunteer and I call on Ellie. Together, this student and I assert that counting "by ones" is not the only way to find the total when presented with an array of objects.

Teaching and Studying Event #10: Teacher Relating the Idea of Groups to Practicing the "Times Tables"

I have made a shift away from the representation of 12 times 22 toward the use of patterns to accomplish calculation. We are still within the domain of giving meaning to multiplication because the particular pattern that this bit of teaching focuses on is doubling the number associated with one group when you can see that two groups have the same number of objects. I will switch again, in the same utterance, to teaching students that counting by sixes is a rehearsal of the six times table. I will then switch yet again, back to the language of grouping, to teach the class that the times tables are about counting *groups* of objects. The patterns in the numbers and in the way we talk about operations on numbers are verbalized and available for study. Also available is a different participation structure: chorusing a simple answer to a simple question. This mode of participating makes it possible for students to try out their thinking in another kind of medium than the single-student answer.

Lampert: Now, let's count by sixes here.

[pointing to the circled 6s] Six, twelve, eighteen, twenty-four, thirty, thirty-six, forty-two, forty-eight, fifty-four, sixty [students counting with Lampert].

That's our six times table. One group of six.

Eddie and Awad you should be looking up here [circling my finger around one 6].

One group of six is six.

Two groups of six is—[circling my finger around two 6s]

Students: Twelve.

Lampert: Three groups of six is—[circling my finger around three 6s]

Students: Eighteen.

Lampert: Four groups of six is—[circling my finger around four 6s]

Students: Twenty-four.

Lampert: Five groups of six is—[circling my finger around five 6s —the whole "top" row]

Students: Thirty.

Lampert: Thirty, and now I can do the same thing. If this much is thirty [again circling my finger around five 6s], how much is the whole amount [circling my finger around both rows of five 6s] going to be? Leticia?

Sixty.

So I have ten groups of six here.

I use the blackboard representation and point with my finger to show a connection between what we say and the pictures of groups of objects. And within this switch, there is another topic to study: the six times table itself, and the importance of developing familiarity with that particular set of "ordered triples" [i.e., (1, 6, 6), (2, 6, 12), (3, 6, 18), (4, 6, 24), and so on.] There is also a social switch away from Richard, and a cognitive switch, away from reasoning to remembering. What I do next with students is to practice associating the numbers in those triples and fix them in everyone's auditory memory by chorusing. I assume that students vary in their facility to recite the six times table and that such chorusing would help to move everyone up a notch on the memory scale.

Teaching and Studying Event #11: Teacher Again Asking for an Explanation

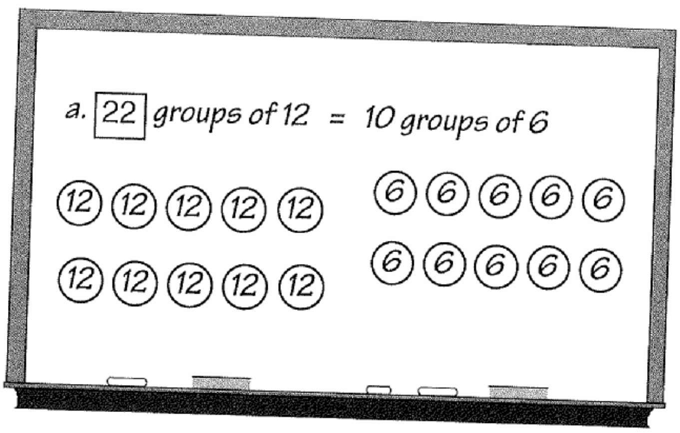
All of this time, as I had been teaching the whole class various aspects of multiplication, I also had been trying to give Richard a way to make sense of his assertion, which was still up on the board: "22 groups of 12 = 10 groups of 6." I focus again on questioning this particular student, while attending to the class in the background.

Lampert:	Now, Richard, what do you think about this twenty-two groups of twelve thing? [several seconds of silence]
	What if I had just ten groups of twelve? How many would that be?
Richard:	I don't know.

At the time, it is unclear whether Richard has learned something from the discussion. He had earlier said, "I timesed twelve and ten. Twelve times ten equals twenty two." Now he says "I don't know." This could be progress. I had been engaging the class in different activities that would illustrate the connection between "timesing," "multiplying," and "grouping," and wanted to find out now if Richard would see that adding 10 and 12 was not an appropriate action to take to find out the answer to my question. He did not repeat his earlier assertion that "Twelve times ten equals twenty-two." Richard's response suggests that he had learned that as a number to go in the box, "22" did not make sense, but he also learned that he did not know what did make sense.

Teaching and Studying Event #12: Teacher Linking the Explanation to the Public Representation

To make a connection between Richard and the rest of the class that both Richard and I might be able to use, I return to the representation I had been generating earlier and had asked students to complete in their imaginations. I add more circled twelves to the earlier "groups of 12" until what is on the left side of the board is symmetrical with the representation of 10 groups of 6 on the right side.



In addition to teaching Richard how to figure out the answer to my question, what I am doing is designed to teach him that he is capable of figuring it out. I am also teaching the rest of the class that Richard is capable of figuring it out and of talking about his thinking.

Teaching and Studying Event #13: Teacher Representing, Student Asserting

Next I did some of the calculation, scaffolding Richard's participation and focusing him on the "doubling" strategy I had earlier discussed with Ellie.

Lampert:	Okay, let's do the top row [pointing to circles].
	Twelve plus twelve is twenty-four, plus twelve is thirty-six, forty-eight, sixty.
	[pointing to a row of five circled 12s] Now if the top row is sixty, how many am I going to have altogether? Richard?
Richard:	One-twenty.
Lampert:	One-hundred and twenty. Now this is ten groups of twelve. Richard, what do you think about your idea of twenty-two groups of twelve?
Richard:	It's wrong.

Here I am teaching Richard and the rest of the class that it is their job to evaluate their own thinking and that a drawing is a useful tool for doing so.

Teaching and Studying Event #14: Student Evaluating Earlier Assertion

I now ask Richard to position his earlier assertion in relation to his current thinking.

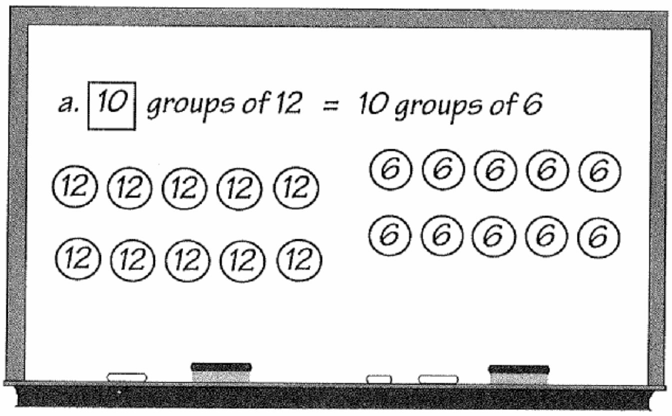
Lampert:	Is it too big or too small?
Richard:	Too big.

As I have been teaching the whole class various aspects of multiplication, I have been trying to give Richard a way to make sense of his assertion, still up on the board. I move back from interacting with other members of the class to questioning him. This part of the journey comes to an end when Richard publicly recants, saying that his idea of 22 groups of 12 is "too big" and that 10 groups of 12 is not right either. By asking Richard to evaluate his answer in terms of too big or too small, I am teaching everyone that getting "in the ballpark" is an appropriate first step toward getting a precise answer.

Teaching and Studying Event #15:
Teacher and Student Reason Collaboratively

In terms of ordinary classroom norms, Richard is being asked to be extraordinarily courageous here. I am demonstrating something to the class using my interaction with him. Because I am teaching him in such a public way, I need to manage the problem of helping him save face with the rest of the class even though he has publicly admitted that he was "wrong." Gingerly, I stick with him, believing perhaps that if I can elicit some publicly respectable reasoning from Richard, I will be able to rescue something of his image as a mathematical problem solver. I reason for and with Richard that if 22 is too big, then what goes in the box needs to be "lower" than 22, giving him some language to continue his nascent reasoning process.

Lampert: Okay, so we have to make this [pointing to the number in the box] lower. Can it be ten [replacing the 22 in the box with a 10]?



To model the "guess and check" strategy, I deliberately choose a lower number that does not fulfill the conditions to solve the equation and address a question about whether "it can be ten" to the whole class. Minimal reasoning is required to conclude that 10 groups of 12 cannot equal 10 groups of 6, but reasoning is required. When I put the "10" in the box, Richard said "no," and others chimed in as well, possibly indicating some camaraderie.